UNIVERSITY OF MANITOBA
Term Test 2D
COURSE: MATH 1500
DATE & TIME: November 1, 2018, 5:40PM – 6:40PM
CRN: various
DURATION: 1 hour EXAMINER: various

INSTRUCTIONS

I. No texts, notes, or other aids are permitted. There are no calculators, cellphones or electronic translators permitted.

II. This exam has a title page, 5 pages of questions and two blank pages for rough work. Please check that you have all the pages. **DO NOT REMOVE THE SCRAP PAPER**

III. The value of each question is indicated in the lefthand margin beside the statement of the question. The total value of all questions is 45 points.

IV. **Answer all questions on the exam paper** in the space provided beneath the question. Unjustified answers will receive little or no credit. Only techniques taught in this course should be used. **Do not continue on the back of the page.** If you need more space, continue on one of the scrap pages, CLEARLY INDICATING THAT YOUR WORK IS TO BE CONTINUED.

V. Do not deface the QR - code in the top right corner. Doing so may result in the page not being scanned and therefore not graded.

Question	Points	Score
1	12	
2	7	
3	5	
4	4	
5	8	
6	9	
Total:	45	

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1. Find the derivative of each of the following functions. DO NOT SIMPLIFY YOUR ANSWER AFTER YOU EVALUATE THE DERIVATIVE.

[4] (a) $f(x) = e^{6x} + 9x^{5/9} + (\ln 3)^{-1} - \dfrac{4}{x}$.

Solution: $f'(x) = 6e^{6x} + 5x^{-\frac{4}{9}} + 0 + 4x^{-2}$.

[4] (b) $g(u) = 5^u\,(\tan u - 4\ln u)$.

Solution: $g'(u) = 5^u \ln 5(\tan u - 4\ln u) + 5^u\left(\sec^2 u - \dfrac{4}{u}\right)$.

[4] (c) $h(z) = \dfrac{(4z+3)^2}{5-3z^2}$.

Solution:

$$\begin{aligned} h'(z) &= \frac{((4z+3)^2)'\,(5-3z^2) - (4z+3)^2(5-3z^2)'}{(5-3z^2)^2} \\ &= \frac{(2)(4z+3)(4)\,(5-3z^2) - (4z+3)^2(-6z)}{(5-3z^2)^2}. \end{aligned}$$

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[7] 2. Find the derivative of the following function. Express your final answer as a function of only the variable x. DO NOT SIMPLIFY YOUR ANSWER.

$$f(x) = (x+3)^{2\cos x}.$$

Solution: Let $y = (x+3)^{2\cos x}$. Using logarithmic differentiation, we first take the logarithm of both sides and apply the properties of logarithms before taking the derivative.

$$\begin{aligned} \ln y &= \ln(x+3)^{2\cos x} \\ \ln y &= (2\cos x)\ln(x+3). \end{aligned}$$

Next, we differentiate both sides with respect to x and then solve for $\frac{dy}{dx}$:

$$\begin{aligned} \frac{1}{y}\frac{dy}{dx} &= (-2\sin x)\ln(x+3) + (2\cos x)\left(\frac{1}{x+3}\right) \\ \frac{dy}{dx} &= y\left((-2\sin x)\ln(x+3) + \frac{2\cos x}{x+3}\right) \\ \frac{dy}{dx} &= (x+3)^{2\cos x}\left((-2\sin x)\ln(x+3) + \frac{2\cos x}{x+3}\right). \end{aligned}$$

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3. A particle is moving along the x-axis, and its displacement in meters after t seconds is given by the function $x(t) = t^2 - 9t - 3$.

[2] (a) What are the velocity and acceleration of the particle at any time?

Solution:
Velocity, $v(t) = x'(t) = 2t - 9$ m/s.
Acceleration, $a(t) = x''(t) = 2$ m/s^2.

[3] (b) When is the speed of the particle equal to 3 m/s?

Solution:
The speed of the particle is equal to 3 m/s when,
$|v| = 3 \Longrightarrow |2t - 9| = 3 \Longrightarrow 2t - 9 = 3 \quad \text{or} \quad 2t - 9 = -3 \Longrightarrow t = 6 \quad \text{or} \quad t = 3.$
So, the speed of the particle is equal to 3 m/s when $t = 3$ s and $t = 6$ s.

4. Calculate each limit below. If a limit does not exist, explain why. Show all work. Writing an answer with no justification may not yield any marks.

[1] (a) $\lim_{x \to 0^+} -4 \log_6 x$.

Solution: Since $\lim_{x \to 0^+} \log_6 x = -\infty$ then $\lim_{x \to 0^+} -4 \log_6 x = +\infty$.

[3] (b) $\lim_{x \to 0} \dfrac{\sin(6x)}{\sin(7x)}$.

Solution:

$$\begin{aligned}
\lim_{x \to 0} \frac{\sin(6x)}{\sin(7x)} &= \lim_{x \to 0} \frac{\sin(6x)}{6x} \left(\frac{7x}{\sin(7x)} \right) \left(\frac{6x}{7x} \right) \\
&= \lim_{x \to 0} \frac{\sin(6x)}{6x} \lim_{x \to 0} \left(\frac{7x}{\sin(7x)} \right) \lim_{x \to 0} \left(\frac{6x}{7x} \right) \\
&= (1)(1)(6/7) = \frac{6}{7}.
\end{aligned}$$

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[8] 5. Find an equation of the tangent to the curve defined implicitly by

$$x^2y + xy^3 - 2x = 2$$

at the point $(2, 1)$.

Solution:

Differentiating the given equation with respect to x, we get

$$\begin{aligned} \frac{d}{dx}(x^2y + xy^3 - 2x) &= \frac{d}{dx}(2) \\ x^2\frac{dy}{dx} + 2xy + 3xy^2\frac{dy}{dx} + y^3 - 2 &= 0 \\ \frac{dy}{dx} &= \frac{2 - y^3 - 2xy}{x^2 + 3xy^2} \end{aligned}$$

Let m be the value of $\frac{dy}{dx}$ when $x = 2$ and $y = 1$, then $m = -\frac{3}{10}$.
Therefore, an equation of the tangent is

$$y - 1 = -\frac{3}{10}(x - 2).$$

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[9] 6. A rectangular box is designed so that its length is always exactly twice its height. At a particular moment, the box has length 10 cm, width 6 cm and height 5 cm. At this moment, the length is decreasing at 2 cm/s. What is the rate of change of width at that moment, if the volume of the box is decreasing at 145 cm^3/s? State your final answer in a complete sentence.

Solution: Let l be the length, w the width and h the height. (The students should draw a diagram and label it)
We have $l = 2h$ at all times.
Thus, the volume V is given by

$$V = lwh = \frac{wl^2}{2}.$$

We know that $\frac{dV}{dt} = -145\ \text{cm}^3/\text{s}$ and $\frac{dl}{dt} = -2$ cm/s.
We want to find $\frac{dw}{dt}$ when $l = 10$, $w = 6$, and $h = 5$.
Differentiating gives

$$\frac{dV}{dt} = wl\frac{dl}{dt} + \frac{l^2}{2}\frac{dw}{dt}.$$

Replacing by the given values,

$$-145 = (6)(10)(-2) + \frac{10^2}{2}\frac{dw}{dt} \implies \frac{-145 + 120}{50} = \frac{dw}{dt} \implies \frac{dw}{dt} = -\frac{1}{2}.$$

Therefore, at the particular moment in question, the width of the box is decreasing at the rate of $\frac{1}{2}$ cm/s.

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